AD-A240 243

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

...d to average 1 hour per response including the time for reviewing instructions searching existing data sources gathering and ...ction of information Send comments regarding this burden estimate or any other aspect of this collection of information including ...ervices Directorate for Information Operations and Reports 1215 Jefferson Davis Highway Suite 1204 Arlington VA 22202 4302 ...n Project (0704 0188) Washington DC 20503

	2 REPORT DATE August 1991	3 REPORT TYPE AND DATES COVERED Professional paper
4 TITLE AND SUBTITLE A RE-EXAMINATION OF THE RELATIONSHIP BETWEEN FUZZY SET THEORY AND PROBABILITY THEORY		5 FUNDING NUMBERS PR: ZW40 PE: 0601152N WU: DN300173
6 AUTHOR(S) I. R. Goodman		
7 PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Ocean Systems Center San Diego, CA 92152-5000		8 PERFORMING ORGANIZATION REPORT NUMBER
9 SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Office of Chief of Naval Research Independent Research Programs (IR) OCNR-10P Arlington, VA 22217-5000		10 SPONSORING/MONITORING AGENCY REPORT NUMBER
11 SUPPLEMENTARY NOTES		
12a DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.		12b DISTRIBUTION CODE

DTIC

13 ABSTRACT *(Maximum 200 words)*

Recently, a manner of articles have appeared in newspapers and popular magazines and journals emphasizing a dichotomy in regard to fuzzy sets: the great potential benefit of using an intuitively appealing and simple approach to modeling uncertainties and its almost universal rejection or avoidance by orthodox scientists in the United States trained in probability theory. Thus, once more, it is of some interest to attempt to determine dispassionately what relations, if any, exist between fuzzy sets and probabilities.

Previously, Goodman et al. were the first to point out that rather basic connections do indeed exist between fuzzy set theory and probability theory via *random sets* and their one point coverage functions. (See, e.g. Goodman, "Some new results concerning random sets and fuzzy sets", *info. Sci.* 34, 1984 or Goodman & Nguyen, *Uncertainty Models for Knowledge-Based Systems* (monograph), North-Holland Co., Amsterdam, 1985.) But relatively few individuals have utilized these connections (Dubois & Prade on occasion and Oblow emphasizing full random set representations through his hybrid "O-Theory"), mainly due to the foreboding complex structure of random sets – as compared to the relatively simple form of random variables or random vectors. Even the relatively recent contributions of Lindley, Klir, and others in comparing the roles and game theoretic admissibility properties of fuzzy sets and probabilities do not address the deeper relations between the two areas.

91-10553

Published in *Proceedings of the 29th Annual Bayesian Research Conference*, 1991.

14 SUBJECT TERMS data fusion; conditional event algebra; temporal operators; modal operators		15 NUMBER OF PAGES
		16 PRICE CODE

17 SECURITY CLASSIFICATION OF REPORT	18 SECURITY CLASSIFICATION OF THIS PAGE	19 SECURITY CLASSIFICATION OF ABSTRACT	20 LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAME AS REPORT

UNCLASSIFIED

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NSN 7540-01-280-5500

Standard form 298

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A RE-EXAMINATION OF THE RELATIONSHIP BETWEEN FUZZY SET THEORY AND PROBABILITY THEORY

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A RE-EXAMINATION OF THE RELATIONSHIP BETWEEN FUZZY SET THEORY AND PROBABILITY THEORY

ABSTRACT

Recently, a number of articles have appeared in newspapers and popular magazines and journals emphasizing a dichotomy in regard to fuzzy sets: the great potential benefit of using an intuitively appealing and simple approach to modeling uncertainties and its almost universal rejection or avoidance by orthodox scientists in the United States trained in probability theory. Thus, once more, it is of some interest to attempt to determine dispassionately what relations, if any, exist between fuzzy sets and probabilities.

Previously, Goodman et al. were the first to point out that rather basic connections do indeed exist between fuzzy set theory and probability theory via *random sets* and their one point coverage functions. (See, e.g. Goodman, "Some new results concerning random sets and fuzzy sets", *Info. Sci. 34*, 1984 or Goodman & Nguyen, *Uncertainty Models for Knowledge-Based Systems* (monograph), North-Holland Co., Amsterdam, 1985.) But relatively few individuals have utilized these connections (Dubois & Prade on occasion and Oblow emphasizing full random set representations through his hybrid "O-Theory"), mainly due to the foreboding complex structure of random sets - as compared to the relatively simple form of random variables or random vectors. Even the relatively recent contributions of Lindley, Klir, and others in comparing the roles and game theoretic admissibility properties of fuzzy sets and probabilities do not address the deeper relations between the two areas.

The thrust of this paper is to explore a new connection between fuzzy sets and probability which is related to random sets, but does not require the full specifications involved in using them. In brief, it is not a new idea that any fuzzy set membership function $f_A: X \to [0,1]$, with domain of definition X corresponding to attribute or fuzzy set A, can be characterized completely as the mean function of a collection (not a stochastic process!) of zero-one random variables $(V_{A,x})_{x \in X}$, where $V_{A,x} = 1$ iff x "ϵ" A (or x has attribute A as a property, etc.) and $V_{A,x} = 0$ iff x "$\not\epsilon$" A (or x does not have attribute A), with $E(V_{A,x}) = \text{prob}(V_{A,x} = 1) = f_A(x)$, for all x in X. Obviously, one can estimate $f_A(x)$ here by the standard sampling procedure counting the number of successful "hits" or 1-outcomes relative to a poll of people, population of stochastic entities, etc. At this point, relatively little else has been achieved in the literature, other than interpreting the zero-one random variables as being the indicator functions in some sense of corresponding random sets. However, by simply reversing the 0 and 1 values above - no informational difference occurs - Sklar's Copula Representation Theorem can be used to show that most standard fuzzy set operations and relations indeed have isomorphic counterparts among naturally corresponding ordinary set operations and relations acting upon zero-one random variables. These correspondences can be extended to arbitrary wff's (well formed formula) of fuzzy expressions (still in the first order sense- i.e., having membership values lying in the unit interval), provided that the fuzzy set operators are restricted to min, max, 1-( ), or prod, probsum, 1-( ), for conjunction, disjunction, and negation, respectively.

The import of the above results is that now combination of evidence or data fusion problems involving both linguistic-based and stochastic information can be treated from a universal probabilistic viewpoint, rather than from a universal fuzzy set one, although one can employ the various fuzzy set techniques developed to first model the linguistic information aspect. Finally, a number of open issues are posed for future investigation, without explicit use of the restraining form of random set theory.